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APPROXIMATE CALCULATION OF HYPERSONIC CONICAL FLOW PARAMETERS FOR AIR IN THERMODYNAMIC EQUILIBRIUM

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APPROXIMATE CALCULATION OF HYPERSONIC CONICAL FLOW PARAMETERS FOR AIR IN THERMODYNAMIC EQUILIBRIUM

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SUMMARY

An approximate algebraic solution for equilibrium flow derived from Dorodnitsyn's integral method is used to compute flow parameters for circular cones at zero incidence in air that is free of argon and carbon dioxide. The air between the shock wave and cone surface is assumed to be in local thermal and chemical equilibrium.

Comparison of 44 test cases of the algebraic solution with a numerical integration of the exact Taylor-Maccoll equation gave average errors of less than 1 percent in the velocity, pressure, density, and temperature on the cone surface and in the shock-layer thickness. For these cases the cone semivertex angle ranged from 20° to 50°, the free-stream velocity from about 10,000 to 32,000 feet per second, and the free-stream pressure from 10^{-4} to 10^{-1} atmosphere. The free-stream temperature was held at 491.69° R. Required inputs are the cone semivertex angle and free-stream velocity, pressure, density, and ratio of specific heats. In the program, values of pressure, density, velocity, temperature, and compressibility factor, both behind the shock wave and on the cone surface are computed.

INTRODUCTION

In applying Dorodnitsyn's integral method to nonequilibrium flow over pointed bodies, South (ref. 1) derived an algebraic solution (the first approximation or one-strip solution) valid for frozen or equilibrium conical flow. For a diatomic gas, he has shown that this solution agrees very well with the exact solution for both frozen and vibrational equilibrium (no dissociation) flow. Since a numerical integration of the exact Taylor-Maccoll equation by Romig (ref. 2) was available for comparison, the algebraic solution was programed for the IBM 7090 electronic data processing system in order to see whether this same agreement would be found for air in dissociation equilibrium. The basic thermodynamic data used were those of Hilsenrath and Beckett (ref. 3) as adapted for high-speed computers by Grabau (ref. 4) and E. F. Maher of the U.S. Naval Ordnance Laboratory, White Oak, Md. The author is indebted to Mr. Maher for providing (through private communication) a FORTRAN coding of empirical equations for the thermodynamic properties of air.

SYMBOLS

The standard conditions and conversion factors used in obtaining numerical values are as follows: p_{std}, standard pressure, 2.1162×10^3 lb/sq ft or 1 atmosphere; T_{std}, standard temperature, 491.69° R; ρ_{std}, standard mass density, 8.0422×10^{-2} lb/cu ft; g, gravitational acceleration, 32.174 ft/sec^2; K', conversion factor, $3.9952 \times 10^{-5} \frac{\text{Btu/lb}}{\text{sq ft/sec}^2}$; K", conversion factor, gp_{std}.

H specific enthalpy, Btu/lb

M Mach number

p pressure, atmosphere

T temperature, $^{\circ}$R

$U = \log_{10} \frac{\rho}{\rho_{std}}$

u,v velocity components in x- and y-directions, ft/sec

V total velocity, $\left(u^2 + v^2\right)^{1/2}$, ft/sec

$X = \log_{10} \frac{p}{p_{std}}$

x,y coordinates along and normal to cone surface

Z compressibility factor

β shock-wave angle, deg

γ ratio of specific heats

δ shock-layer thickness normal to body

θ cone semivertex angle, deg

ρ mass density, lb/cu ft

ω shock deflection angle, deg

Subscripts:

o quantity on cone surface

δ quantity behind shock

∞ quantity in free stream

METHOD

Approximate Algebraic Solution

The reader is referred to reference 1 for a detailed description of the method as well as the derivation of the algebraic solution. This solution (ref. 1) is

$$\rho_o u_o + \left[\cot(\beta - \theta) + \cot\theta\right]\rho_\delta v_\delta = 0 \tag{1}$$

$$2\rho_o u_o^2 + 2\left[\cot(\beta - \theta) + \cot\theta\right]\rho_\delta u_\delta v_\delta + K''\left(p_o - p_\delta\right) = 0 \tag{2}$$

$$2\left[\cot(\beta - \theta) + \cot\theta\right]\rho_\delta v_\delta^2 - K''\left[2\cot(\beta - \theta) + \cot\theta\right]\left(p_o - p_\delta\right) = 0 \tag{3}$$

where the angles β and θ are shown in figure 1.

The equations (1) to (3) are obtained from approximate ordinary differential equations which result from integration of the divergence form of the continuity and the two component momentum equations across the shock layer ($y = 0$ to $y = \delta$). If similarity considerations (that is, straight shock wave and constant properties along the surface) are then applied, equations (1) to (3) are obtained and are valid for frozen or equilibrium flow within the approximation. These equations can be manipulated to give u_o as a function of the shock properties and the cone semivertex angle θ:

$$u_o = u_\delta + \frac{v_\delta}{2\cot(\beta - \theta) + \cot\theta} \tag{4}$$

As is pointed out in reference 1, equations (1) to (4) do not depend on any particular thermodynamic model for the gas, since neither the equation of state nor the energy equation has been used.

Thermodynamic Model

In order that a direct comparison of the present solution could be made with that of reference 2, the same basic thermodynamic data were used. The basic data were those of reference 3 for air that is free of argon and carbon dioxide, that is, for a mixture of nitrogen and oxygen where the percentage of each has been increased to compensate for the lack of argon and carbon dioxide. The present program uses these basic data (ref. 3) as adapted for high-speed computers by Grabau (ref. 4) and Maher. Reference 2 used data derived by Blackwell, et al. (ref. 5) from reference 3.

The equation of state and the energy equation are, respectively,

$$\frac{p}{p_\infty} = \frac{Z\rho T}{Z_\infty \rho_\infty T_\infty} \tag{5}$$

$$H_\infty + \frac{K'}{2} V_\infty^2 = H + \frac{K'}{2} V^2 \tag{6}$$

The FORTRAN program of Maher computes Z and H (in Btu/lb) as a function of $X = \log_{10} \frac{p}{p_{std}}$ and $U = \log_{10} \frac{\rho}{\rho_{std}}$. Thus, with the free-stream velocity, pressure, and density specified, the free-stream compressibility factor, temperature, and total enthalpy can be computed.

Equations (1), (2), and (3) were derived (ref. 1) without recourse to the isentropic condition; thus in the method of the present paper, the isentropic condition will not be utilized. However, in reference 2, the Taylor-Maccoll equation was integrated along isentropes in order to have a simple tabulation of the speed of sound.

Oblique-Shock Functions

For an oblique shock wave (inclined at an angle β to the free stream) the conservation of mass and momentum can be written as

$$\rho_\infty V_\infty \sin\beta = \rho_\delta V_\delta \sin(\beta - \omega) \tag{7}$$

$$\rho_\infty V_\infty^2 \sin\beta \cos\beta = \rho_\delta V_\delta^2 \sin(\beta - \omega)\cos(\beta - \omega) \tag{8}$$

$$K''p_\infty + \rho_\infty V_\infty^2 \sin^2\beta = K''p_\delta + \rho_\delta V_\delta^2 \sin^2(\beta - \omega) \tag{9}$$

where ω is the shock deflection angle. (See fig. 1.) Equations (7) to (9) are combined to give

$$\frac{\rho_\delta}{\rho_\infty} = \frac{\tan\beta}{\tan(\beta - \omega)} \tag{10}$$

$$\left(\frac{V_\delta}{V_\infty}\right)^2 = \cos^2\beta + \left(\frac{\rho_\infty}{\rho_\delta}\right)^2 \sin^2\beta \tag{11}$$

$$\frac{p_\delta}{p_\infty} = 1 + \frac{V_\infty^2 \rho_\infty}{K'' p_\infty} \sin^2\beta \left(1 - \frac{\rho_\infty}{\rho_\delta}\right) \tag{12}$$

It is seen from figure 1 that

$$u_\delta = V_\delta \cos(\theta - \omega) \tag{13}$$

$$v_\delta = -V_\delta \sin(\theta - \omega) \tag{14}$$

Computational Procedure

If the cone semivertex angle θ is specified, equations (1), (3), (4), (10), (11), (12), (13), and (14) give properties as a function of β, ω, and the free-stream conditions. The quantities β and ω are determined when the energy equation (eq. (6)) is satisfied both behind the shock wave and on the cone surface. This is accomplished by successive guesses on β and ω for the given input conditions. The steps of the procedure are as follows:

(1) An initial value of β is computed from a simple approximate frozen solution derived by Hord (ref. 6):

$$(\beta)_{\text{initial}} = \text{arc cos}\left(1 - \frac{\gamma_\infty + 1}{2} \sin^2\theta - \frac{1}{M_\infty^2}\right)^{1/2} \tag{15}$$

where γ_∞ is the free-stream ratio of specific heats and M_∞ is computed from V_∞, p_∞, ρ_∞, and γ_∞. (Note that γ_∞ is only required as an input to start the iterative scheme.)

(2) For convenience, an initial value of $\frac{\rho_\delta}{\rho_\infty}$ was selected rather than a value for ω. (See eq. (10).) For this initial value, the frozen density ratio given in reference 7 as

$$\frac{\rho_\delta}{\rho_\infty} = \frac{\left(\gamma_\infty + 1\right) M_\infty^2 \sin^2\beta}{\left(\gamma_\infty - 1\right) M_\infty^2 \sin^2\beta + 2} \tag{16}$$

is used.

(3) The values of V_δ and p_δ are calculated from equations (11) and (12), respectively, and H_δ is computed from U_δ and X_δ.

(4) The energy equation (6) is tested by using shock variables. If it does not balance (to a relative error of 10^{-5} in H_δ), ρ_δ is adjusted and step (3) is repeated, β being held fixed. When equation (6) balances, ω is calculated from equation (10).

(5) Values of u_δ, v_δ, u_o, ρ_o, and p_o are calculated from equations (13), (14), (4), (1), and (3), respectively, and H_o is computed from U_o and X_o.

(6) The energy equation (eq. (6)) is tested by using cone surface variables. If it does not balance (to a relative error of 10^{-5} in H_o), β is changed and the process is repeated from step (2).

The entire iteration employs neither the equation of state (eq. (5)) nor the temperature explicitly. The compressibility and temperature both behind the shock and on the cone surface are calculated after the iteration is completed. Numerical values of constants used in the program are given under Symbols.

RESULTS AND DISCUSSION

For 44 cases computed with the approximate algebraic solution, the initial conditions were selected so that a direct comparison could be made with the results of reference 2. For these cases the cone semivertex angle ranged from 20° to 50°, the free-stream velocity from about 10,000 to 32,000 feet per second, and the free-stream pressures from 10^{-4} to 10^{-1} atmosphere. The free-stream temperature was held at 491.69° R. As indicated in reference 2, the equilibrium assumption should be valid for normal free-stream velocities ($V_\infty \sin \beta$) greater than 10,000 feet per second and for free-stream pressures larger than 10^{-4} atmosphere. Table I gives the average and maximum percentage deviations of various calculated quantities (denoted by F) from those of Romig (ref. 2).

TABLE I.- PERCENTAGE DEVIATION OF CALCULATED QUANTITIES

FROM THOSE OF REFERENCE 2

$$\left[\text{Percent deviation of } F \equiv \left|\frac{(F)_{calc} - (F)_{Romig}}{(F)_{Romig}}\right|(100)\right]$$

	Percent deviation[1] of -					
	β	$\beta - \theta$	u_o	p_o	ρ_o	T_o
Average deviation	0.061	0.802	0.034	0.704	0.699	0.453
Maximum deviation	.147	1.274	.174	1.697	1.467	.968

[1]Based on 44 cases.

Even though the average expected error in the thermodynamic data used in the IBM program (refs. 3, 4, and Maher) was about 2 percent, the approximate solution consistently gave a computed shock angle about 0.1 percent larger and a cone surface velocity 0.1 percent smaller than those obtained in reference 2. All other quantities agreed, on the average, to within 1 percent with a spread on both sides of the value given in reference 2. This consistent deviation in the case of β and u_o is, therefore, that due to the approximate solution. This deviation is in agreement with the results of reference 1 (see table I) where a consistent error of less than 0.1 percent and a larger shock thickness were found when the approximate solution was compared with the exact Taylor-Maccoll solution for nitrogen in vibrational equilibrium.

The total time required on an IBM 7090 electronic data processing system to obtain the solutions for the preceding 44 cases was 1.3 minutes; each case was computed from a separate set of initial parameters.

For the free-stream conditions, $p_\infty = 10^{-3}$ atmosphere and $T_\infty = 491.69^\circ$ R, a number of cases were computed for cones with semivertex angles of 20°, 30°, 40°, and 50°. These results are given in figures 2 to 6 in order to show the comparison with the results of reference 2.

Shown in figures 2 and 3 are, respectively, the angular shock-layer thickness $\beta - \theta$ and the ratio of cone surface to free-stream velocity $\frac{u_o}{V_\infty}$ as a function of free-stream Mach number. The agreement for these two quantities appears to be very good.

In reference 2, correlation of properties with respect to the hypersonic similarity parameter $M_\infty \sin \theta$ has been discussed. Shown in figures 4 to 6 are the variations of the thermodynamic ratios $\frac{p_o}{p_\infty}$, $\frac{T_o}{T_\infty}$, and $\frac{\rho_o}{\rho_\infty}$ with respect to the hypersonic similarity parameter $M_\infty \sin \theta$. The solid curve in these figures represents the average value of a thermodynamic ratio as a function of $M_\infty \sin \theta$, whereas the circles give the average value of $M_\infty \sin \theta$ (ref. 2) for given thermodynamic ratios. The agreement with reference 2 is seen to be good.

Several hand calculations were made with the approximate solution. It was found that a case could be computed in about 3 hours. For these calculations, the thermodynamic data of reference 3 were plotted on large sheets of semilog paper so that an accuracy of approximately 2 percent could be obtained.

CONCLUDING REMARKS

An approximate algebraic solution for equilibrium flow derived from Dorodnitsyn's integral method has been used to compute flow parameters for circular cones at zero incidence in air that is free of argon and carbon dioxide. The air between the shock wave and cone surface is assumed to be in local thermal and chemical equilibrium.

Comparison of all cases with a numerical integration of the exact Taylor-Maccoll equation gave average deviations of about 1 percent; thus, it appears that the first integral approximation will handle cases involving dissociation equilibrium phenomena fairly accurately.

Secondly, the approximate algebraic solution was found to provide a method of computing hypersonic conical flow parameters rapidly. This method should be of great value for design type calculations.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., October 11, 1963.

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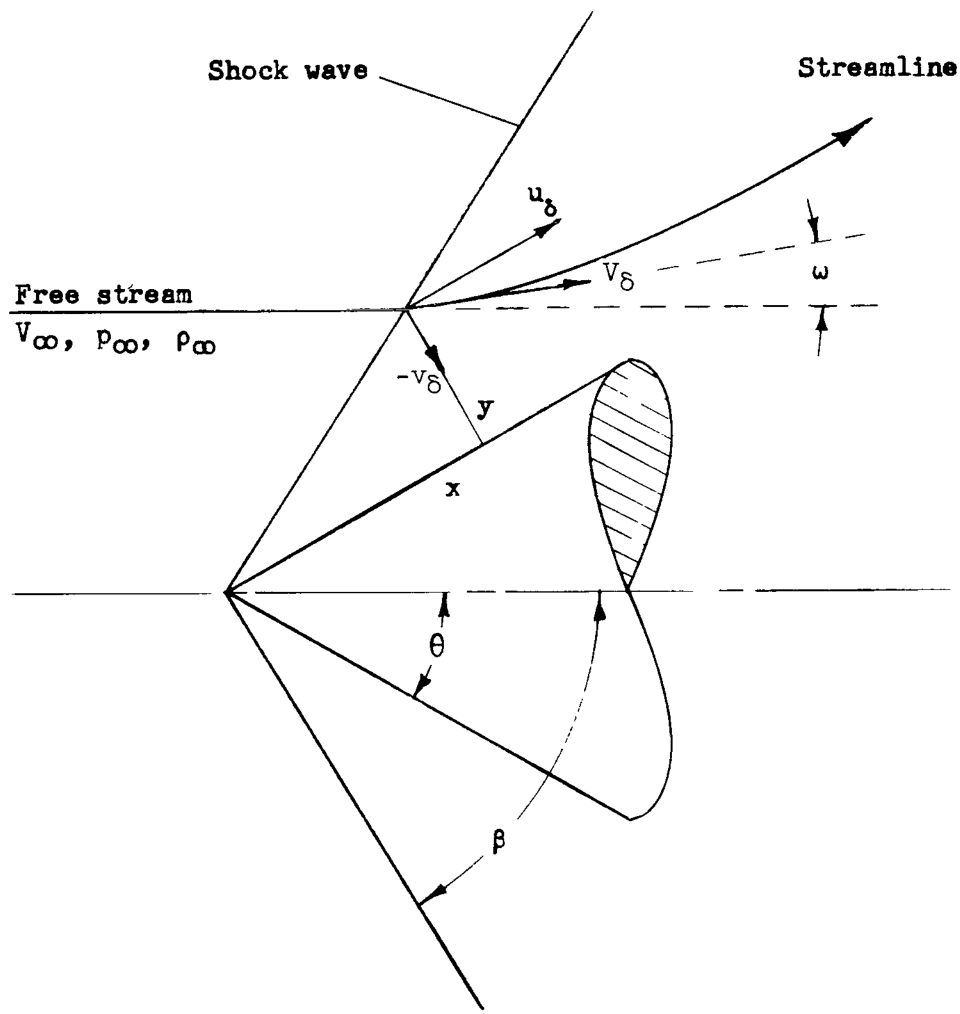

Figure 1.- Geometry and coordinate system.

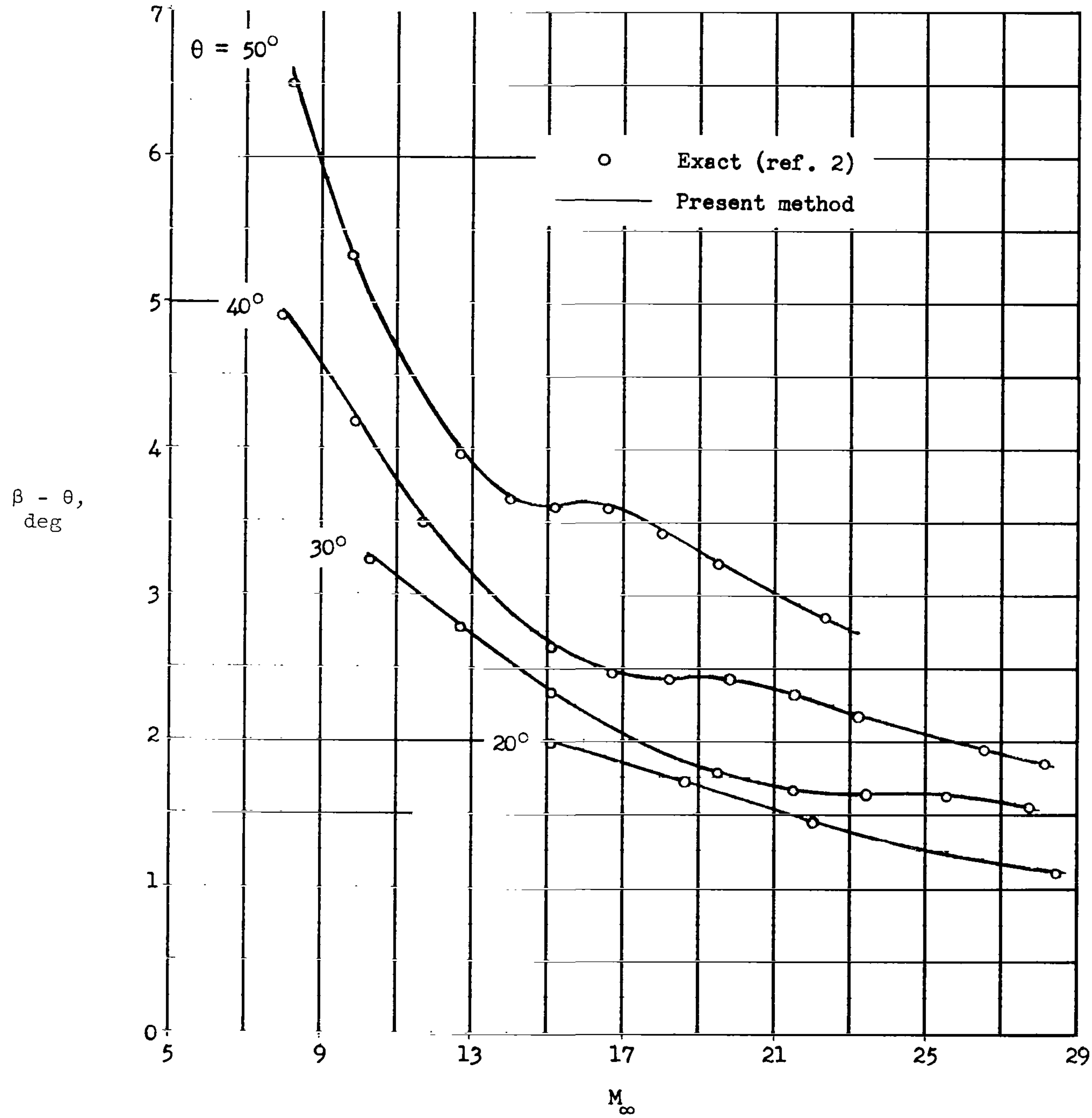

Figure 2.- Variation of angular shock-layer thickness with free-stream Mach number. $p_\infty = 10^{-3}$ atmosphere; $T_\infty = 491.69^\circ$ R.

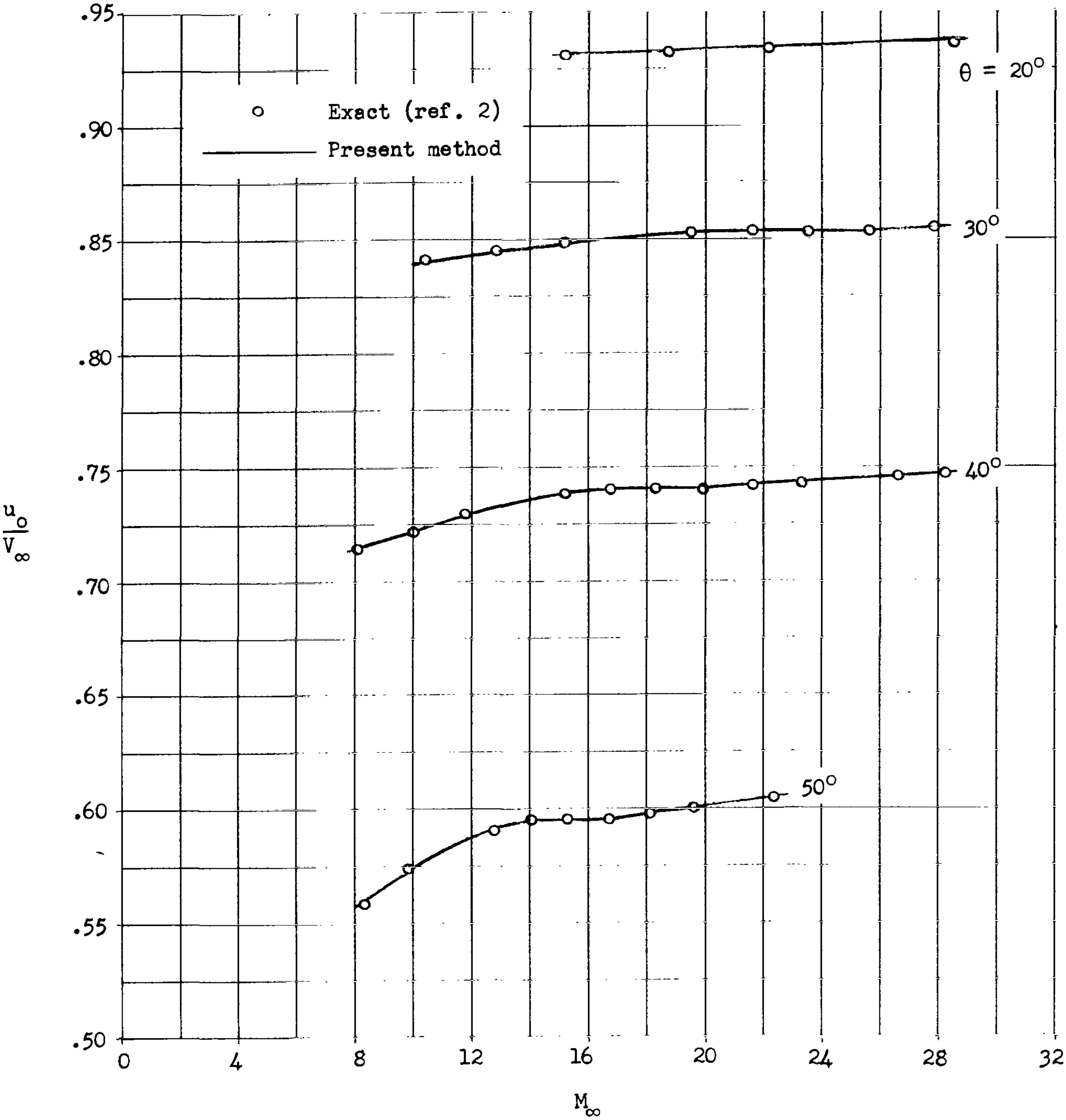

Figure 3.- Variation of the ratio of cone surface velocity to free-stream velocity with free-stream Mach number. $p_\infty = 10^{-3}$ atmosphere; $T_\infty = 491.69^\circ$ R.

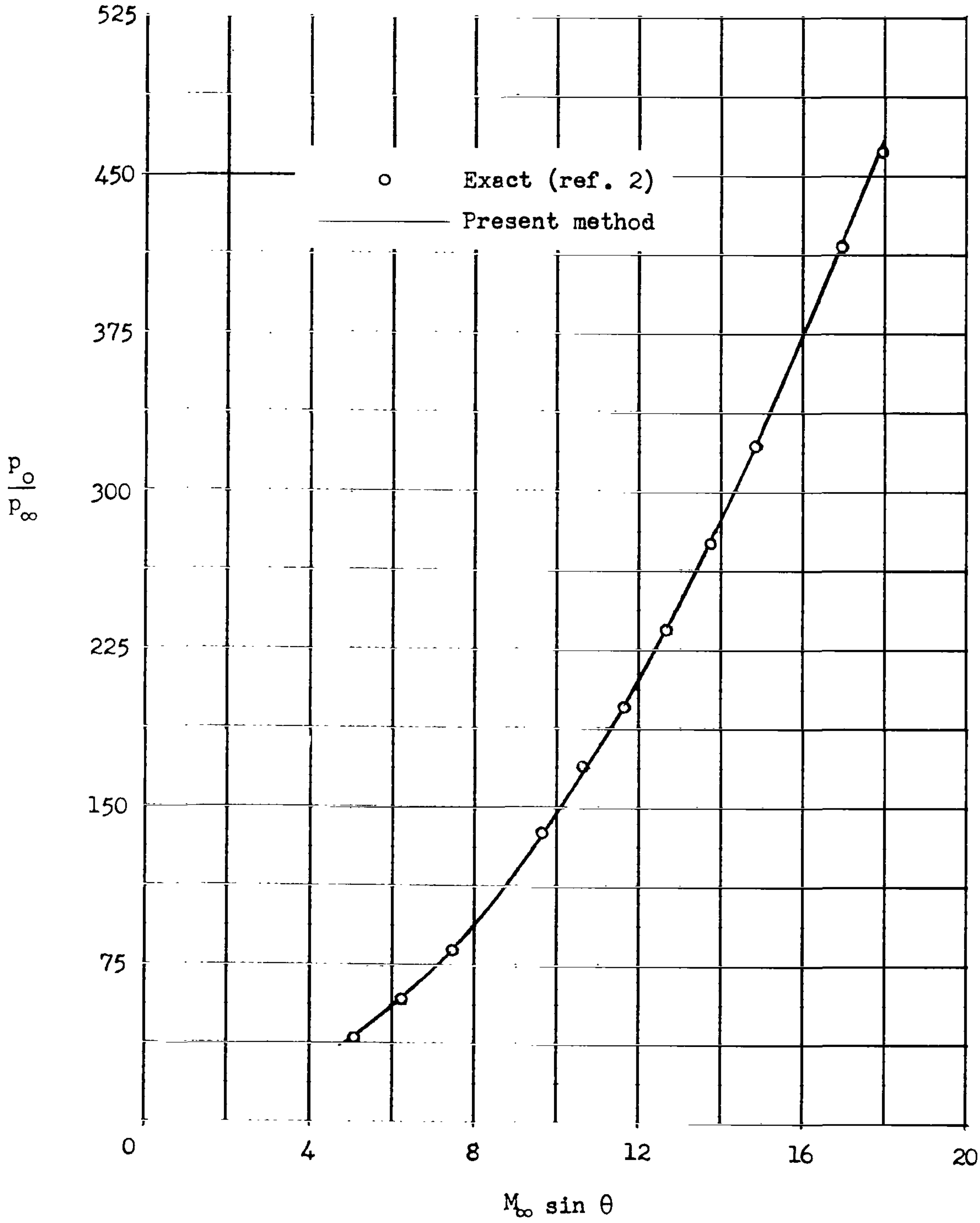

Figure 4.- Variation of the ratio of cone surface pressure to free-stream pressure with hypersonic similarity parameter. $p_\infty = 10^{-3}$ atmosphere; $T_\infty = 491.69^\circ$ R.

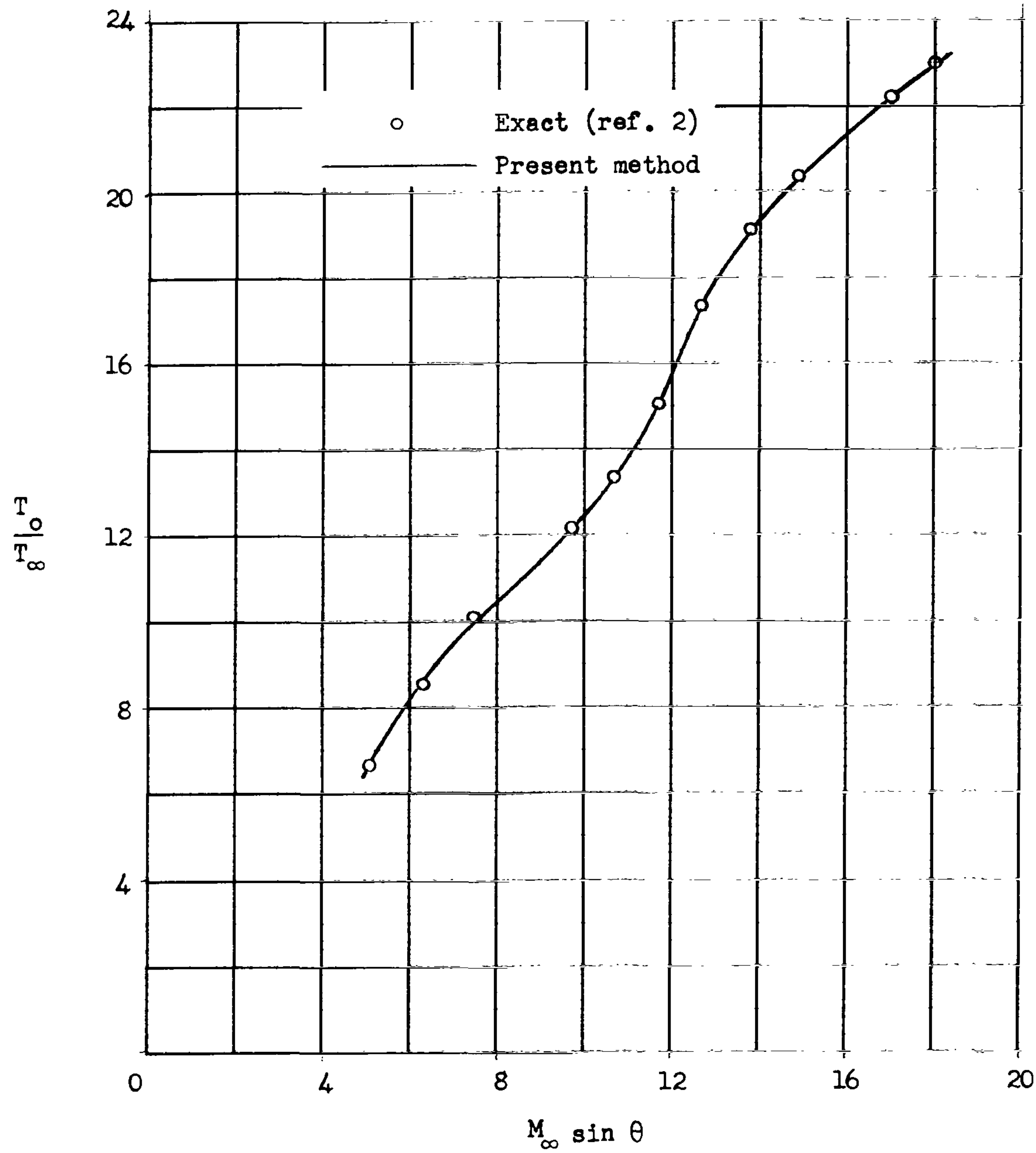

Figure 5.- Variation of ratio of cone surface temperature to free-stream temperature with hypersonic similarity parameter. $p_\infty = 10^{-3}$ atmosphere; $T_\infty = 491.69^\circ$ R.

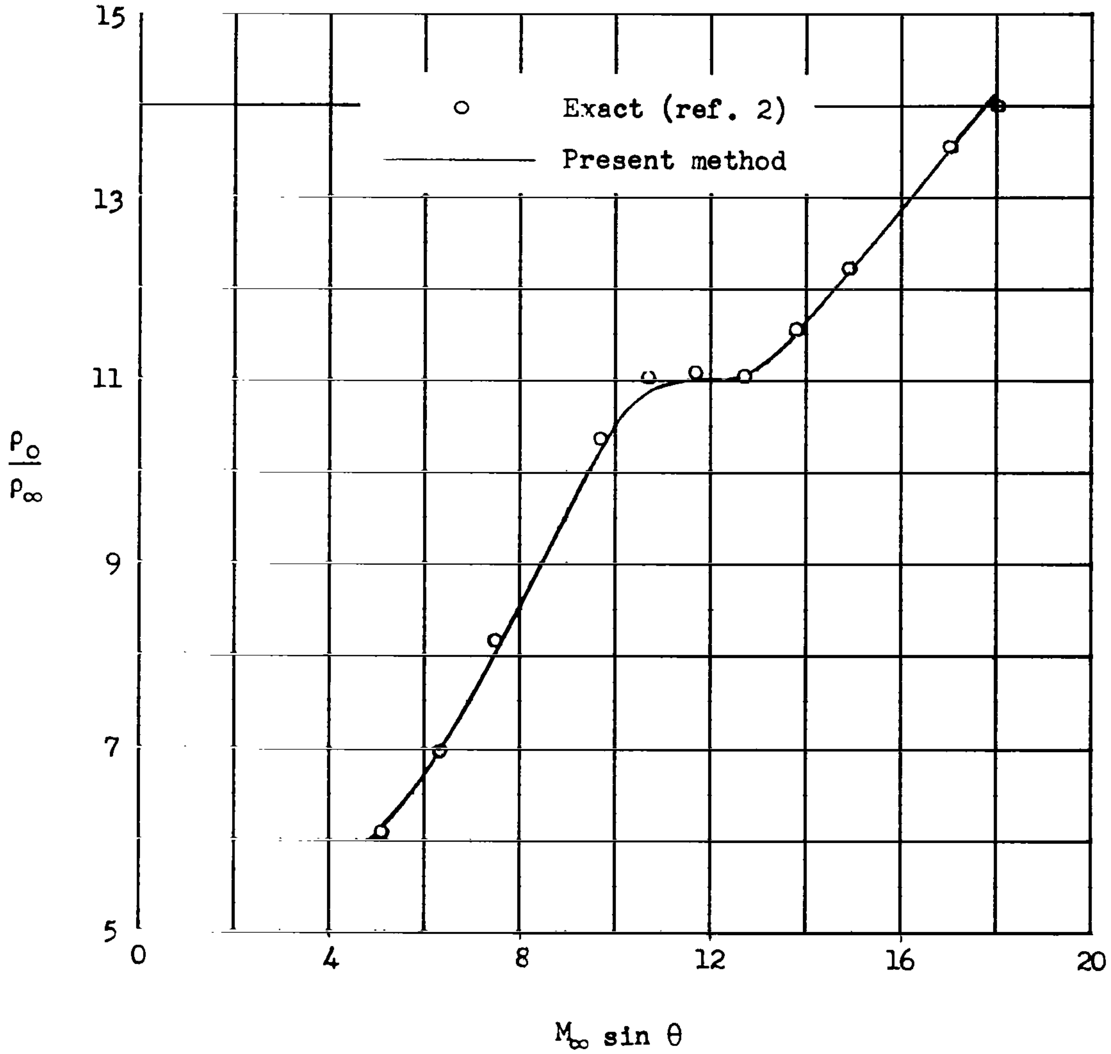

Figure 6.- Variation of ratio of cone surface density to free-stream density with hypersonic similarity parameter. $p_\infty = 10^{-3}$ atmosphere; $T_\infty = 491.69^\circ$ R.